Permutations of $\{1, 2, 3, 4\}$ (the symmetric group S_4)

This applet can be used either to express one permutation as a product of permutations of a particular kind, such as transpositions, or to express the composite of two, or more, permutations in its cycle decomposition.

It may be used to demonstrate

- how to compose permutations;
- that every permutation can be expressed as a product of transpositions;
- to explain and illustrate standard formulae for expressions as in (b);
- that every transposition can be expressed as a product of adjacent transpositions or transpositions of the form $(1\ j)$;
- that every even permutation can be expressed as a product of cycles of length three;
- the impossibility of expressing an odd permutation as a product of cycles of length three;
- various sets of generators for the groups S_4 and A_4;
- conjugates in S_4 (though a separate applet for this is under development).

In its most basic role, the applet is effectively a calculator in S_4. Two or more permutations can be selected from an array showing all 24 elements of S_4 and then composed, with the composition displayed. Alternatively, one permutation can be set as a target to be expressed as a composite of permutations of a particular type.

Navigation

- The initial display shows the 24 elements of S_4 in an array, each with a selection button.
- To compose two or more permutations, select them from the array, in the order in which they are to be applied; the effect of each will be displayed and the permutations will appear in the panel above the array.
- When all the permutations to be composed have been entered, click successively on **Compose**; the cycle decomposition of the composite will be displayed one symbol at a time.
- To express one permutation as a product of permutations of a given type, select the permutation from the array, click **Target**, and select the type from the drop-down menu **Available permutations**. The array will be restricted to permutations of the required type.
- Select potential factors in the order in which they are to be applied; successive compositions will be displayed and, optionally, a joyful message should greet achievement of the target (see configurability).

Examples 1. To express (1 4 3 2) as a product of three transpositions.

1. From the array, select (1 4 3 2) by clicking on the circle next to it.
2. Click on **Target**.
3. Click on **Available permutations** and, from the drop-down menu, choose **transpositions**.
4. Select a first move from those available, perhaps by aiming to get a particular symbol in its target position.
5. Aim to achieve the target by applying two more transpositions.

2. To find the cycle decomposition of (1 4 3 2)(3 4).

1. Click **reset** if you have already used the applet for another calculation.
2. Select (3 4) from the panel, and then select (1 4 3 2).
3. Click on **compose** until the full cycle decomposition appears.
4. The intermediate display (14 shows that 1 is sent to 4 by the composite; the next click reveals that 4 is sent to 2 and this can be seen by tracking the 4th symbol ♣ through the display.

Configurability From the html file , the following can be configured:

1. Whether permutations are composed from the right $((i)\alpha\beta = ((i))\alpha)\beta)$ or the left $(\alpha\beta(i) = \alpha(\beta(i)))$. Set the parameter *right* to *yes* or *no*.
2. Whether a message is shown whenever the target is achieved. Set the parameter *alert* to *yes* or *no*.
3. Whether composites are computed one element at a time, by successive clicks, or by a single click. Set the parameter *live* to *yes* (for successive clicks) or *no*.
4. Colours used for background, foreground (text) and highlighted text (these can also be set within the applet); the parameters "background", "foreground" and "highlight" can each be set to any of b(blue), r(red), g(green), y(yellow), c(cyan), p(pink), w(white), 0(black), n(navy).
5. The font size (this can also be set within the applet); the parameter "size" can be set to any of 1(normal), 2(large), 3(huge).