

Powers/Orders of Permutations

This applet demonstrates powers of permutations in S_n , with $n \leq 9$. It is helpful in introducing the idea of order for a permutation or for group elements in general. It demonstrates

- the repeating patterns in powers of permutations or group elements of finite order;
- cycle decompositions of powers of a given permutation;
- that a cycle of length n has order n ;
- that the order of a product of disjoint cycles is the least common multiple of the lengths of the cycles;
- the significance of the word "disjoint" in the previous item.

A permutation is entered using a drop-down menu, featuring some preset permutations and a user-defined option, under which any permutation in S_9 can be entered. As a side product, this applet will also work out the cycle decomposition of a product of non-disjoint cycles such as $(2\ 4)(1\ 2\ 3)(4\ 5)(1\ 2)(3\ 4\ 5)$. As an aid to understanding this process, the applet *S4Applet* is better, but it is restricted to S_4 .

Navigation

- The dropdown menu for selecting permutations is labelled (123), one of the preset options. One can select one of the preset options or **user**.
- When **user** is selected, (1) appears in a panel to the left and can be edited to enter any permutation in S_9 as a product of cycles by editing the default (1).
- As soon as a cycle decomposition is entered, clicking the Apply button will display the effect of the permutation.
- Subsequent clicking of Apply displays successive powers, up to the 21st, 21 being one more than the maximum order of an element of S_9 .
- A counter indicates the most recently displayed power and the screen should scroll automatically.
- An option (see configurability) allows for a message to be shown when the most recently displayed power is the identity.

Example To display the 4th power of the 8-cycle $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)$ as a product of 4 disjoint transpositions:

1. Click on the (123) button and, from the drop-down menu, choose **user**;
2. In the panel to the left of this button, type in the amendments needed to change (1), which appears automatically, to (12345678).
3. Click on **apply**; the cycle is applied once.

4. Repeat three times until the counter shows 4; the effect of the 4th power of the cycle on eight letters is displayed and, on the left, the cycle decomposition of the 4th power is shown.
5. Higher powers may be shown by further clicking of **Apply**, including the repeating pattern beyond the eighth power.

Configurability From the html file , the following can be configured:

1. Whether permutations are composed from the right ($((i)\alpha\beta = ((i)\alpha)\beta$) or the left ($(\alpha\beta(i) = \alpha(\beta(i)))$). Set the parameter *right* to *yes* or *no*.
2. Whether a message is shown whenever the displayed power is the identity. Set the parameter *alert* to *yes* or *no*.
3. The message shown when, in (ii), *alert* is set to *yes*. Edit the parameter *alertmessage* to give the required text.
4. Colours used for background, foreground (text) and highlighted text (these can also be set within the applet). The parameters "background", "foreground" and "highlight" can each be set to any of b(blue), r(red), g(green), y(yellow), c(cyan), p(pink), w(white), 0(black), n(navy).
5. font size (this can also be set within the applet). The parameter "size" can be set to any of 1(normal), 2(large), 3(huge).