Orbits and Stabilizers

This applet demonstrates orbits and stabilizers of colourings of a square grid for the action of the group of symmetries of the square. It is helpful in illustrating the orbit-stabilizer theorem. It demonstrates

- orbits for particular colourings;

- stabilizers for particular colourings;

- how orbits and stabilizers measure symmetry;

- informally, the inverse proportionality between orbits and stabilizers;

- formally, the orbit-stabilizer theorem with each element of an orbit corresponding to a coset of the stabilizer.

A square grid is coloured, using a palette, and the symmetries of the square are applied to generate the orbit. This is displayed initially by group element, but may be rearranged so that different group elements giving the same colouring are displayed together.

Navigation

- The size of grid, 3 by 3, 4 by 4 (default) or 5 by 5, is selected using the **Size of Grid** button.

- To colour a square in a chosen colour, select the colour from the palette and click on the square.

- For a colouring in which a particular colour dominates, the whole grid can be coloured using the **Fill** button.

- Once the grid is fully coloured, the **Orbit** button displays the eight colourings obtained by applying symmetries of the grid, each labelled by the appropriate group element.

- The **Arrange** button, rearranges the colourings to show each distinct colouring corresponding to a coset of the stabilizer.

- The **Reset** button restores the original screen.

Example To show the symmetries of an $\ulcorner$-shaped colouring:

1. Select 3×3 from the dropdown menu on the **Size of Grid** button.

2. Click on a colour on the palette and click on the **Fill** button.

3. Click on a second colour on the palette and click , successively, on each square in the top row or left hand column of the grid.

4. Click on **Orbit** to generate eight colourings; each of $\ulcorner, \urcorner, \llcorner, \lrcorner$ is obtained once by a rotation (top row of display) and once by a reflection (second row).

5. Click on **Arrange** to rearrange the colourings so that each pair of identical colourings appears adjacently, with the elements of the corresponding coset of the stabilizer displayed below each pair.

Configurability From the html file , the following can be configured:

1. The preferred names for the symmetries of the square are set using the parameters *gr1* to *gr8*.

2. Colours used for background, foreground (text) and highlighted text (these can also be set within the applet). The parameters *background*, *foreground* and *highlight* can each be set to any of b(blue), r(red), g(green), y(yellow), c(cyan), p(pink), w(white), 0(black), n(navy).

3. The font size (this can also be set within the applet). The parameter *size* can be set to any of 1(normal), 2(large), 3(huge).